DJJ-003-016404 Seat No. ________

M. Sc. (Sem. IV) (Maths) (CBCS) Examination

May / June – 2015

CMT-4004 : Graph Theory

Faculty Code : 003
Subject Code : 016404

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Each question carries equal marks.
(2) Attempt all the questions.

1 Choose appropriate alternative : (any **seven**) **7×2=14**

(1) Let G be a null graph. What is sum of degree of all vertices $\sum_{v \in V(G)} d(v)$ for G ?

(A) 0 (B) $|V(G)|$

(C) 1 (D) $2|V(G)|$

(2) Let $G=(V,E)$ be a disconnected graph and $K =$ number of components of G. Then which of following is true for G ?

(A) $k=1$ (B) $k \le |V|$

(C) $k > |V|$ (D) None of these

(3) Which of following is not an Euler graph ?

(A) C_{99}

(B) K_{99}

(C) P_{99}

(D) 4-regular connected graph

(4) Which of following is not a Hamiltonian graph ?

(A) C_9 (B) C_{10}

(C) K_{10} (D) P_{11}

(5) What is the number of vertices in a tree with 20 edges ?

(A) 20 (B) 19

(C) 21 (D) 40

(6) Let $G=(V,E)$ be a connected graph with $|V|=10$ and $|E|=20$. What is number of edges in a spanning tree T for G ?

(A) 10 (B) 20

(C) 19 (D) 9

(7) Which of following is not a planar graph ?

(A) K_6 (B) P_{19}

(C) C_{20} (D) $K_{2,3}$

(8) Let $G=(V,E)$ be a connected planar graph with 10 vertices and four faces (regions). What is number of edges in the graph G ?

(A) 14 (B) 12

(C) 9 (D) 40

2 Attempt any two : **2×7=14**

(1) Let $G=(V,E)$ be a graph. Let $x, y \in V$ be such that $d_G(x)$ and $d_G(y)$ both are odd. If $d_G(w) = \text{even}$, $\forall\, w \in V - \{x, y\}$, then prove that there must be a path between x and y in G.

(b) Let $G=(V,E)$ be a finite graph. Then prove that there are subgraphs $G_1, \ldots\ldots, G_k$ of G such that $G_i = (V_i, E_i), \forall\, i = 1, 2, \ldots\ldots, k$ and

(i) Each G_i is maximal connected subgraph of G.

(ii) $V_i \cap V_j = \phi,\ \forall\, i, j \in \{1, \ldots\ldots, k\}$ and $i \neq j$

(iii) $V = V_1 \cup V_2 \cup \ldots\ldots \cup V_k,\ \ E = E_1 \cup E_2 \cup \ldots\ldots \cup E_k$.

(c) Let $G=(V,E)$ be a connected graph. Prove that there is a spanning tree $T=(T,F)$ for G.

(d) Let $G=(V,E)$ be a connected graph. Prove that G is a tree *iff* adding an edge between any two vertices of G creates exactly one circuit.

3 Attempt any one : **1×14=14**

(a) State and prove the Euler's theorem.

(b) Let $G=(V,E)$ be a connected graph. Prove that ring sum of two distinct cut-sets of G is either a cut-set of G or it is an edge disjoint union of two cut-sets of G.

(c) Let $G=(V,E)$ be a simple graph with $n=|V|\geq 3$. If $d_G(v)\geq \frac{n}{2}$, $\forall\, v\in V$ then prove that G is a Hamiltonian graph.

4 Attempt any **two** : **2×7=14**

(a) Define closure of a graph G. For a simple graph G, prove that G is Hamiltonian *iff* its closure $C(G)$ is Hamiltonian.

(b) Let u, v be two distinct vertices of a tree T. Prove that there is a unique path between u and v in T.

(c) Define minimally connected graph G. Prove that a connected graph G is minimally connected graph *iff* it is a tree.

(d) Define incidence matrix $A(G)$ for a graph G. Also write atleast five properties for the incidence matrix $A(G)$.

5 Attempt any seven : **7×2=14**

(1) Write adjacency matrix $X(G)$, where $G=C_3$, cycle on three vertices.

(2) Define chromatic number for a graph G. Give an example of a graph G which is a 1-chromatic graph.

(3) Define diametrer of a connected graph and draw a connected graph G whose diameter $D(G) = 3$.

(4) Draw a simple connected graph $G = (V, E)$ with $|V| = 5$ and G is a Hamiltonian graph as well as G is an Eulerian graph.

(5) Define a connected graph and draw a disconnected graph G with $|V(G)| = 3$.

(6) Define a self loop and parallel edges in a graph G.

(7) Give definition : Regular graph, Simple graph.

(8) Define isomorphism of graphs.